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MATHEMATICAL MODELING OF GENERALIZED BOUNDARY VALUE NEYMAN HEAT EXCHANGE BLANK PIECE-WISE HOMOGENEOUS CYLINDER

Abstract. The mathematical model of temperature distribution in an empty piecewise uniform cylinder, which rotates at a constant angular velocity axis OZ taking into account the finite speed of propagation of heat in the form of Neumann boundary problem of mathematical physics. Developed new integral transformation for piecewise homogeneous space with which found the temperature field empty piecewise homogeneous circular cylinder in a convergent series in orthogonal functions and Fourier Bessel.

Keywords: Neumann boundary value problem, generalized equation of energy transfer, integrated Laplace, Fourier, relaxation time.

Introduction. In the phenomenological theory of heat conduction is assumed that the rate of heat distribution is infinite [1,2]. However, high-intensive non-stationary processes occur, such as explosions, supersonic flow, high speeds of rotation using this assumption leads to errors, so be aware that the heat is spread with the ultimate speed.

As the heat literature review in the cylinders that rotate, studied at present is not enough [3,4]. It was shown [1] that numerical methods unsteady heat transfer problems neosesymetrychnyh cylinders that rotate are not always effective when it comes to computing at high speeds. So have [1], which provided stability calculations in finite element method and finite difference method used to calculate unsteady temperature fields neosesymetrychnyh cylinders that rotate are determined similar characteristics. These conditions are as follows:

$$1-\frac{\Delta F_0}{\Delta\varphi^2}\geq 0 \quad \text{and} \quad \frac{1}{\Delta\varphi}-\frac{Pd}{2}\geq 0,$$

where F_0 - Fourier criterion; Pd - the criterion Predvoditeleva.

If $Pd=10^5$, corresponding to the angular velocity of the metal cylinder $\omega=1{,}671\,сек^{-1}$ radius of 100 mm, variable $\Delta\varphi$ and ΔF_o should be subject to the following conditions:

$$\Delta\varphi\leq 2\cdot 10^{-5} \quad \text{and} \quad \Delta F_o\leq 2\cdot 10^{-10}.$$

To evenly cooled cylinder provided Bi = 5 time required to 90% temperature reached steady state, equal to $F_o \approx 0.025$ [1]. This means that you need to make at least $1.3 \cdot 10^8$ operations on time in order to achieved steady temperature distribution.

Moreover, it should be noted that during one cycle of calculations required to make $3.14 \cdot 10^5$ computing, since internal state of ring $3.14 \cdot 10^5$ is characterized points. The result shows that the number of calculations required for numerical result seems unrealistic.

Therefore, to solve the boundary problem that arises in the mathematical modeling of unsteady heat transfer process in the cylinder, rotating integral transformations will apply.

The aim is to develop a new generalized mathematical model of temperature distribution in a piecewise homogeneous cylinder as Neumann boundary problem of mathematical physics for the heat equation, and solve the resulting boundary problem, the solutions which are used in the fields of temperature control.

Main part. Consider the calculation of unsteady temperature field empty circular cylinder outer radius R in a cylindrical coordinate system (r, φ, z), piecewise uniform in the direction of the polar radius r, which rotates at a constant angular velocity axis OZ, taking into account the finite speed of propagation of heat. Thermal properties are in each layer independent of temperature conditions ideal thermal contact between the layers and internal heat source available. At the initial time constant temperature of the cylinder G_0, and the outer and inner surfaces of the cylinder are known heat flows $G(\varphi)$ i $G_1(\varphi)$ respectively.

$\theta(\rho, \varphi, t)$ relative temperature of the cylinder can be written as:

$$\theta(\rho,\varphi,t) = \begin{cases} \theta_1(\rho,\varphi,t) & \text{if} \quad \rho \in (\rho_0, \rho_1) \\ \theta_2(\rho,\varphi,t) & \text{if} \quad \rho \in (\rho_1, \rho_2) \end{cases} \tag{1}$$

Relative temperature $\theta_s(\rho, \varphi, t)$ s th layer cylinder calculated by the formulas:

$$\theta_s(\rho,\varphi,t) = \frac{T_s(\rho,\varphi,t) - G_0}{T_{max} - G_0},$$

where $T_s(\rho, \varphi, t)$ - temperature s th layer cylinder; T_{max} -The maximum temperature of the cylinder; $\rho = \dfrac{r}{R}$; s = 1,2.

In [1] obtained generalized equation for energy transfer element driving continuum, taking into account the finite velocity of heat distribution. According to [1] generalized equation of energy balance of solids, which rotates at a constant angular velocity axis OZ, thermal properties which are independent of temperature, and no internal heat source takes the form:

$$\gamma\,\mathrm{c}\left\{\frac{\partial \mathrm{T}}{\partial \mathrm{t}}+\omega\frac{\partial T}{\partial \varphi}+\tau_r\left[\frac{\partial^2 T}{\partial t^2}+\omega\frac{\partial^2 T}{\partial \varphi \partial t}\right]\right\}=\lambda\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}+\frac{\partial^2 T}{\partial z^2}\right], \qquad (2)$$

where γ – density of the medium; c-specific heat; λ - thermal conductivity; $T(\rho,\varphi,t)$-temperature environment; t -time; τ_r -time relaxation.

Mathematically, the problem of determining the relative temperature of the cylinder $\theta(\rho,\varphi,t)$ consists in integrating hyperbolic partial differential equations of heat conduction (2) in $D_s=\{(\rho,\varphi,t)|\rho\in(\rho_{s-1},\rho_s),\varphi\in(0,2\pi),\mathrm{t}\in(0,\infty)\}$, that, given the assumptions adopted in written form:

$$\frac{\partial \theta_s}{\partial t}+\omega\frac{\partial \theta_s}{\partial \varphi}+\tau_r\frac{\partial^2 \theta_s}{\partial t^2}+\tau_r\omega\frac{\partial^2 \theta_s}{\partial \varphi \partial t}=\alpha_s^2\left[\frac{\partial^2 \theta_s}{\partial \rho^2}+\frac{1}{\rho}\frac{\partial \theta_s}{\partial \rho}+\frac{1}{\rho^2}\frac{\partial^2 \theta_s}{\partial \varphi^2}\right] \qquad (3)$$

with initial conditions

$$\theta_s(\rho,\varphi,0)=0, \quad \frac{\partial \theta_s(\rho,\varphi,0)}{\partial t}=0 \qquad (4)$$

boundary conditions

$$\int_0^t \left.\frac{\partial \theta_1}{\partial \rho}\right|_{\rho=\rho_0} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=W(\varphi), \qquad \int_0^t \left.\frac{\partial \theta_2}{\partial \rho}\right|_{\rho=\rho_2} e^{\frac{\zeta-t}{\tau_r}}\,d\zeta=V(\varphi) \qquad (5)$$

conditions ideal thermal contact

$$\theta_1(\rho_1,\varphi,t)=\theta_2(\rho_1,\varphi,t) \qquad (6)$$

$$\lambda_1\frac{\partial \theta_1(\rho_1,\varphi,t)}{\partial \rho}=\lambda_2\frac{\partial \theta_2(\rho_1,\varphi,t)}{\partial \rho} \qquad (7)$$

where $\rho_1=\frac{R_1}{R}$; $\rho_0=\frac{R_0}{R}$; $\rho_2=1$; R_0 - inner radius of the cylinder; R_1 - radius limits layers; λ_s - coefficient of thermal conductivity, γ_s – density, c_s -pytoma heat capacity, $a_s=\frac{\lambda_s}{c_s\gamma_s}$ - thermal diffusivity s-th layer cylinder; $\alpha_s^2=\frac{a_s}{R^2}$; s = 1,2;

$$W(\varphi)=\frac{G_1(\varphi)\tau_r}{\lambda_1(T_{\max}-G_0)};\ V(\varphi)=\frac{G(\varphi)\tau_r}{\lambda_2(T_{\max}-G_0)}\ ;G_1(\varphi),\ G(\varphi)\in \mathrm{C}(0,2\pi).$$

Then the solution boundary value problem (3) - (7) $\theta_s(\rho,\varphi,t)$ is twice continuously differentiated by ρ, φ, t in D and continuous in $\overline{D}$ [5], that is,

$\theta_s(\rho,\varphi,t)\in C^{2,1}(D)\bigcap C(\overline{D})$, and the function $W(\varphi)$, $V(\varphi)$, $\theta_s(\rho,\varphi,t)$, can be arranged in a complex Fourier series [5]:

$$\begin{Bmatrix} \theta_s(\rho,\varphi,t) \\ W(\varphi) \\ V(\varphi) \end{Bmatrix} = \sum_{n=-\infty}^{+\infty} \begin{Bmatrix} \theta_{s,n}(\rho,t) \\ W_n \\ V_n \end{Bmatrix} \cdot \exp(in\varphi), \tag{8}$$

where

$$\begin{Bmatrix} \theta_{s,n}(\rho,t) \\ V_n \\ V_n \end{Bmatrix} = \frac{1}{2\pi} \int_0^{2\pi} \begin{Bmatrix} \theta_s(\rho,\varphi,t) \\ W(\varphi) \\ V(\varphi) \end{Bmatrix} \cdot \exp(-in\varphi) d\varphi ,$$

where $\theta_{s,n}(\rho,t) = \theta_{s,n}^{(1)}(\rho,t) + \mathrm{I}\,\theta_{s,n}^{(1)}(\rho,t)$; $\mathrm{V_n} = V_n^{(1)} + I\,\mathrm{V_n^{(2)}}$; $\mathrm{W_n} = W_n^{(1)} + I\,\mathrm{W_n^{(2)}}$; .I– imaginary unit.

Given that the actual function $\theta_s(\rho,\varphi,t)$, we restrict further consideration $\theta_{s,n}(\rho,t)$ for n = 0,1,2, ..., because $\theta_{s,n}(\rho,t)$ and $\theta_{s,-n}(\rho,t)$ be complex conjugated [5]. Substituting the values of functions (8) to (3) - (7) we obtain a system of differential equations:

$$\frac{\partial \theta_{s,n}^{(i)}}{\partial t} + \vartheta_n^{(i)} \theta_{s,n}^{(m_i)} + \tau_r \frac{\partial^2 \theta_{s,n}^{(i)}}{\partial t^2} + \tau_r \vartheta_n^{(i)} \frac{\partial \theta_{s,n}^{(m_i)}}{\partial t} = \alpha_s^2 \left[\frac{\partial^2 \theta_{s,n}^{(i)}}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta_{s,n}^{(i)}}{\partial \rho} - \frac{n^2}{\rho^2} \theta_{s,n}^{(i)} \right] \tag{9}$$

with initial conditions

$$\theta_{s,n}^{(i)}(\rho,0) = 0, \qquad \frac{\partial \theta_{s,n}^{(i)}(\rho,0)}{\partial t} = 0 \tag{10}$$

boundary conditions

$$\int_0^t \frac{\partial \theta_{1,n}^{(i)}}{\partial \rho}\Bigg|_{\rho=\rho_0} e^{\frac{\zeta-t}{\tau_r}} d\zeta = W_n^{(i)}, \qquad \int_0^t \frac{\partial \theta_{2,n}^{(i)}}{\partial \rho}\Bigg|_{\rho=\rho_2} e^{\frac{\zeta-t}{\tau_r}} d\zeta = V_n^{(i)} \tag{11}$$

conditions ideal thermal contact

$$\theta_{1,n}^{(i)}(\rho_1,t) = \theta_{2,n}^{(i)}(\rho_1,t) \tag{12}$$

$$\lambda_1 \frac{\partial \theta_{1,n}^{(i)}(\rho_1,t)}{\partial \rho} = \lambda_2 \frac{\partial \theta_{2,n}^{(i)}(\rho_1,t)}{\partial \rho} \tag{13}$$

where $\vartheta_n^{(1)} = -\omega n$; $\vartheta_n^2 = \omega n$; $\mathrm{m_1} = 2$, $\mathrm{m_2} = 1$; ; i,s=1,2.

We use the system of differential equations (10) with boundary and initial conditions (10) - (13) integral Laplace transform [6]:

$$\tilde{f}(s)=\int_0^{\infty} f(\tau)e^{-s\tau}\,\mathrm{d}\tau\,.$$

As a result, we obtain a system of ordinary differential equations for $\tilde{\theta}_{s,n}^{(i)}$:

$$s\tilde{\theta}_{s,n}^{(i)}+\vartheta_n^{(i)}\left(\tilde{\theta}_{s,n}^{(m_i)}+\tau_r s\tilde{\theta}_n^{(m_i)}\right)+\tau_r s^2\tilde{\theta}_{s,n}^{(i)}=\alpha_s^2\left[\frac{d^2\tilde{\theta}_{s,n}^{(i)}}{d\rho^2}+\frac{1}{\rho}\frac{d\tilde{\theta}_{s,n}^{(i)}}{d\rho}-\frac{n^2}{\rho^2}\tilde{\theta}_{s,n}^{(i)}\right] \quad (14)$$

boundary conditions

$$\left.\frac{\partial\tilde{\theta}_{s,n}^{(i)}}{\partial\rho}\right|_{\rho=0}=\tilde{W}_n^{(i)},\qquad \left.\frac{\partial\tilde{\theta}_{s,n}^{(i)}}{\partial\rho}\right|_{\rho=1}=\tilde{V}_n^{(i)}, \quad (15)$$

conditions ideal thermal contact

$$\tilde{\theta}_{1,n}^{(i)}(\rho_1,t)=\tilde{\theta}_{2,n}^{(i)}(\rho_1,t) \quad (16)$$

$$\lambda_1\frac{\partial\tilde{\theta}_{1,n}^{(i)}(\rho_1,t)}{\partial\rho}=\lambda_2\frac{\partial\tilde{\theta}_{2,n}^{(i)}(\rho_1,t)}{\partial\rho} \quad (17)$$

where $\tilde{W}_n^{(i)}=W_n^{(i)}\left(1+\frac{1}{s\tau_r}\right)$; $\tilde{V}_n^{(i)}=V_n^{(i)}\left(1+\frac{1}{s\tau_r}\right)$; (i=1,2).

To solve the boundary problem (14) - (17) integral transformation construct:

$$\bar{f}(\mu_{n,k})=\int_{\rho_0}^{\rho_2}\frac{Q_0(\mu_{n,k}\rho)}{\alpha(\rho)}\,\rho\,\mathrm{f}(\rho)d\rho\;=\;\sum_{s=1}^{2}\int_{\rho_{s-1}}^{\rho_s}\frac{Q_s(\mu_{n,k}\rho)}{\alpha_s^2}\,\rho\,\mathrm{f}(\rho)d\rho\,, \quad (18)$$

where $Q_0(\mu_{n,k}\rho)$, $\alpha(\rho)=\begin{cases}Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right), & \alpha_1^2 \quad \text{if} \quad \rho\in(\rho_0,\rho_1)\\ Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right), & \alpha_2^2 \quad \text{if} \quad \rho\in(\rho_1,\rho_2)\end{cases}$.

$Q_0(\mu_{n,k}\rho)$ eigenfunctions and eigenvalues are $\mu_{n,k}$ with the decision of the Sturm-Liouville:

$$\frac{d^2Q_s}{d\rho^2}+\frac{1}{\rho}\frac{dQ_s}{d\rho}-\frac{n^2}{\rho^2}+\frac{\mu^2{}_{n,k}}{\alpha_s^2}Q_s=0 \quad (19)$$

$$\frac{\partial Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)}{\partial\rho}=0\,,\qquad \frac{\partial Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)}{\partial\rho}=0\,, \quad (20)$$

$$Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)=Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right),\ \ \lambda_1\ \frac{\partial Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\partial\rho}=\lambda_2\frac{\partial Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\partial\rho},\ (\text{ s=1,2; })\quad(21)$$

Solving the problem of Sturm-Liouville (19) - (21) we get:

$$Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)=\frac{\Lambda\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)}{\Lambda\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)},\qquad Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)},\quad(22)$$

де $\Lambda\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_1}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)J_n\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)Y_m\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)\right];$

$\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_2}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J_n\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y_m\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\right];$

$J_n(x),Y_n(x)$–Bessel functions $1^{го}$ і $2^{го}$ kind $n^{го}$ in accordance with the procedure[5].

The eigenvalues are $\mu_{n,k}$ solution of the transcendental equation:

$$\frac{\Omega\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}{\Lambda\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}=\sigma\frac{H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)},\quad(23)$$

де$\Omega\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_1}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)J_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)Y_n'\left(\frac{\mu_{n,k}}{\alpha_1}\rho\right)\right];$

$H\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)=\frac{\mu_{n,k}}{\alpha_2}\left[Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)-J_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)Y_n'\left(\frac{\mu_{n,k}}{\alpha_2}\rho\right)\right];\ \sigma=\frac{\lambda_2}{\lambda_1}.$

Inverse transformation formula is:

$$\mathrm{f}(\rho)=\sum_{n=1}^{\infty}\frac{Q_0(\mu_{n,k}\rho)}{\left\|Q_0(\mu_{n,k}\rho)\right\|^2}\ \bar{f}(\mu_{n,k})\quad(24)$$

Foursquare own rules $\left\|Q_0(\mu_{n,k}\rho)\right\|^2$ function is:

$$\left\|Q_0\left(\mu_{n,k}\rho\right)\right\|^2 = \sum_{s=1}^{2}\frac{1}{\alpha_s^2}\int_{\rho_{s-1}}^{\rho_s}\left[Q_s\left(\mu_{n,k}\rho\right)\right]^2\rho\, d\rho = \frac{\rho_1^2}{2\alpha_1^2}\left\{\left(1-\frac{\mathrm{n}^2\alpha_1^2}{\mu_{n,k}^2\rho_1^2}\right)+\left[\frac{\alpha_1\Omega\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\mu_{n,k}\Lambda\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2\right\}-\frac{\rho_0^2}{2\alpha_1^2}\cdot$$

$$\left(1-\frac{\mathrm{n}^2\alpha_1^2}{\mu_{n,k}^2\rho_0^2}\right)\left[\frac{\Lambda\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)}{\Lambda\left(\frac{\mu_{n,k}}{\alpha_1}\rho_1\right)}\right]^2+\frac{\rho_2^2}{2\alpha_2^2}\left(1-\frac{\mathrm{n}^2\alpha_2^2}{\mu_{n,k}^2\rho_2^2}\right)\left[\frac{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_2\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2-\frac{\rho_1^2}{2\alpha_2^2}\left\{\left(1-\frac{\mathrm{n}^2\alpha_2}{\mu_{n,k}^2\rho_1^2}\right)+\right.$$

$$\left.\left[\frac{H\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}{\Psi\left(\frac{\mu_{n,k}}{\alpha_2}\rho_1\right)}\right]^2\right\}.$$

We use the system of differential equations (14) integral transformation (18), where their function $Q_s\left(\frac{\mu_{n,k}}{\alpha_s}\rho\right)$ determined by formulas (22) and the eigenvalues are $\mu_{n,k}$ solution of transcendental equation (23) and given the designation (1), as a result of the petitions system conventional algebraic equations $\overline{\overline{\theta}}_n^{(i)}$:

$$s\overline{\overline{\theta}}_n^{(i)}+\vartheta_n^{(i)}\left(\overline{\overline{\theta}}_n^{(m_i)}+\tau_r.s\overline{\overline{\theta}}_n^{(m_i)}\right)+\tau_r.s^2\overline{\overline{\theta}}_n^{(i)}=q_{n,k}\left(\frac{\Omega_{\mathrm{n,k}}^{(\mathrm{i})}}{\mu^2{}_{n,k}}-\overline{\overline{\theta}}_n^{(i)}\right) \qquad (25)$$

де $q_{n,k}=\mu^2{}_{n,k}$;. $\Omega_{\mathrm{n,k}}^{(\mathrm{i})}=\rho_0 Q_1\left(\frac{\mu_{n,k}}{\alpha_1}\rho_0\right)\widetilde{W}_n^{(i)}+Q_2\left(\frac{\mu_{n,k}}{\alpha_2}\right)\widetilde{V}_n^{(i)}$; i=1,2.

Solving the system of equations (25) we get:

$$\widetilde{\overline{\theta}}_n^{(i)}=\frac{\Omega_{\mathrm{n,k}}^{(\mathrm{i})}\left(\tau_r.s^2+s+\mathrm{q_{n,k}}\right)+(-1)^{i+1}\omega n\widetilde{V}_n^{(m_i)}\Omega_{\mathrm{n,k}}^{(m_i)}\left(1+s\tau_r\right)}{\left(\tau_r.s^2+s+\mathrm{q_{n,k}}\right)^2+\omega^2n^2\left(1+s\tau_r\right)^2}. \qquad (\mathrm{i}=1,2) \qquad (26)$$

Using the image features (26) Formula inverse Laplace obtain original features:

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$$\bar{\theta}_n^{(1)}\left(\mu_{n,k},t\right)=\sum_{j=1}^{2}\zeta_{n,k}\left(s_j\right)\cdot\left\{\Omega_{\mathrm{n,k}}^{(1)}\left(s_j\right)\cdot\left[\left(2\tau_r s_j+1\right)+\tau_r\omega nI\right]+\Omega_{\mathrm{n,k}}^{(2)}\left(s_j\right)\cdot\left[\tau_r\omega n-\left(2\tau_r s_j+1\right)I\right]\right\}\cdot$$
$$\left(e^{s_j t}-1\right)+\sum_{j=3}^{4}\zeta_{n,k}\left(s_j\right)\cdot\left\{\Omega_{\mathrm{n,k}}^{(1)}\left(s_j\right)\cdot\left[\left(2\tau_r s_j+1\right)-\tau_r\omega nI\right]+\Omega_{\mathrm{n,k}}^{(2)}\left(s_j\right)\cdot\left[\tau_r\omega n+\left(2\tau_r s_j+1\right)I\right]\right\}$$
$$\cdot\left(e^{s_j t}-1\right), \tag{27}$$

$$\bar{\theta}_n^{(2)}\left(\mu_{n,k},t\right)=\sum_{j=1}^{2}\zeta_{n,k}\left(s_j\right)\cdot\left\{\Omega_{\mathrm{n,k}}^{(2)}\left(s_j\right)\cdot\left[\left(2\tau_r s_j+1\right)+\tau_r\omega nI\right]-\Omega_{\mathrm{n,k}}^{(1)}\left(s_j\right)\cdot\left[\tau_r\omega n-\left(2\tau_r s_j+1\right)I\right]\right\}\cdot$$
$$\left(e^{s_j t}-1\right)+\sum_{j=3}^{4}\zeta_{n,k}\left(s_j\right)\cdot\left\{\Omega_{\mathrm{n,k}}^{(2)}\left(s_j\right)\cdot\left[\left(2\tau_r s_j+1\right)-\tau_r\omega nI\right]-\Omega_{\mathrm{n,k}}^{(1)}\left(s_j\right)\cdot\left[\tau_r\omega n+\left(2\tau_r s_j+1\right)I\right]\right\}$$
$$\left(e^{s_j t}-1\right), \tag{28}$$

where $\zeta_{n,k}\left(s_j\right)=\dfrac{0.5s_j^{-1}}{\left(2\tau_r s_j+1\right)^2+\left(\tau_r\omega n\right)^2}$, and the value s_j for j = 1,2,3,4 determined by formulas

$$s_{1,2}=\frac{\left(\tau_r\omega ni-1\right)\pm\sqrt{\left(1+\tau_r\omega ni\right)^2-4\tau_r q_{n,k}}}{2\tau_r},\ s_{3,4}=\frac{\left(\tau_r\omega ni+1\right)\pm\sqrt{\left(1-\tau_r\omega ni\right)^2-4\tau_r q_{n,k}}}{2\tau_r}.$$

Thus considering the inverse transformation formulas (8) and (24) we obtain the temperature field piecewise homogeneous circular cylinder towards the polar radius that rotates at a constant angular velocity ω axis OZ, taking into account the finite speed of propagation of heat:

$$\theta\left(\rho,\varphi,t\right)=\sum_{n=-\infty}^{+\infty}\left\{\sum_{\mathrm{k}=1}^{\infty}\left[\bar{\theta}_n^{(1)}\left(\mu_{n,k},t\right)+I\cdot\bar{\theta}_n^{(2)}\left(\mu_{n,k},t\right)\right]\frac{Q_0\left(\mu_{n,k}\rho\right)}{\left\|Q_0\left(\mu_{n,k}\rho\right)\right\|^2}\right\}\cdot\exp\left(in\varphi\right), \tag{29}$$

where value $\bar{\theta}_n^{(1)}\left(\mu_{n,k},t\right)$ and $\bar{\theta}_n^{(2)}\left(\mu_{n,k},t\right)$ determined by formulas (27) (28).

Conclusions. In the article the developed new integral transformation found temperature field (29), empty piecewise homogeneous circular cylinder towards the polar radius that rotates at a constant angular velocity ω axis OZ, taking into account the finite speed of propagation of heat in the form of convergent orthogonal series by Bessel functions and Fourier. Found analytical solution of the generalized boundary problem of heat transfer cylinder that rotates, given finite velocity of propagation of heat can be used in the modulation of temperature fields arising in many technical systems (in satellites, rolls, turbines, etc.).

REFERENCES

1. Berdnyk M. G. Matematychne modeljuvannja tryvymirnoi uzagal'nenoi zadachi teploobminu sucilnogo cylindra, jakyj obertajet'sja /Berdnyk M. G. //Pytannja prykladnoi' matematyky i matematychnogo modeljuvannja. –2014.–S. 26-35.

2. Konet I. M. Giperbolichni krajovi zadachi v neobmezhenyh trysharovyh oblastjah / I. M. Konet, M. P. Lenjuk. – Lviv, 2011. – 48 s. – (Prepr./ NAN Ukrainy In-t prykladnyh problem mehaniky i matematyky im. Ja. S. Pidstrygacha; 01.11).

3. Golicyna E. V. Matematicheskoe modelirovanie temperaturnogo polja v polom vrashhajushhemsja cilindre pri nelinejnyh granichnyh uslovijah / E.V. Golicyna // Teplofizika vysokih temperatur. Nojabr'-Dekabr'. – 2008. – tom 46, № 6. – C. 905 – 910.

4. Gromyk A. P. Nestacionarni zadachi teploprovidnosti v kuskovo-odnoridnyh prostorovyh seredovyshhah / A. P. Gromyk, I. M. Konet. – Kam'janec'-Podil's'kyj : Abetka-Svit. – 2009. – 120 s.

5. Markovych B. M. Rivnjannja matematychnoi fizyky / Markovych B. M. Lviv: Vydavnyctvo Lvivskoi politehniky. - 2010. - 384 c.

6. Lopushans'ka G.P. Peretvorennja Furje, Laplasa: uzagalnennja ta zastosuvannja /G.P. Lopushanska, A.O., Lopushanskyj, O.M. Mjaus. – Lviv.: LNU im. Ivana Franka. - 2014. - 152 s.

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Бердник М.Г. **Математичне моделювання задачі теплообміну рідини на гідродинаміческі початковій ділянці** // Системні технології. Регіональний міжвузівський збірник наукових праць. – Випуск 1(100). – Дніпропетровськ, 2017. – С. - .

Розроблена математична модель температурних розподілів у порожньому кусково-однорідному циліндрі, який обертається з постійною кутовою швидкістю навколо осі OZ з урахуванням кінцевої швидкості поширення тепла, у вигляді крайової задачі Неймана математичної фізики . Розроблено нове інтегральне перетворення для кусково-однорідного простору, за допомогою якого знайдено температурне поле порожнього кусково-однорідного кругового циліндра у вигляді збіжних ортогональних рядів по функціям Бесселя і Фур'є.

Ключові слова: крайова задача Неймана, узагальнене рівняння переносу енергії, інтегральні перетворення Лапласа, Фур'є, час релаксації.

Бібл. 6.

536.24

Бердник М.Г. **Математическое моделирование обобщенной краевой задачи Нейман теплообмена полого кусочно-однородного цилиндра** // Системные технологии; Региональный межвузовский сборник научных работ. – Выпуск (100). – Днепропетровск, 2016. – С.

Разработана математическая модель температурных распределений в полом кусочно-однородном цилиндре, который вращается с постоянной угловой скоростью вокруг оси OZ, с учетом конечной скорости распространения тепла в виде краевой задачи Неймана математической физики. Разработано новое интегральное преобразование для кусочно-однородного пространства, с помощью которого найдено температурное поле пустого кусочно-однородного кругового цилиндра в виде сходящихся ортогональных рядов по функциям Бесселя и Фурье.

Ключевые слова:краевая задача Неймана, обобщенное уравнение переноса энергии, интегральные преобразования Лапласа, Фурье, время релаксации..

Библ. 6.

UDC 536.24
Berdnyk M. **Mathematical modeling of heat transfer fluid in the initial section hydrodynamycheskye** // System technologies. – N.1 (100). – Dnipropetrovs’k, 2016. – P. .

A mathematical model of the temperature distribution in the hollow piecewise uniform cylinder, which rotates at a constant angular velocity about the axis OZ, taking into account the finite speed of propagation of heat in the form of the Neumann boundary problem of mathematical physics. Created a new integral transform of a piecewise-homogeneous space, with which found the temperature field empty piecewise homogeneous circular cylinder in the form of convergent orthogonal series of Bessel functions and of Fourier.

Keywords: Neumann boundary value problem, generalized equation of energy transfer, integrated Laplace, Fourier, relaxation time**.**
Bibl. 6.

АВТОРСЬКА ДОВІДКА

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